Design and Stress Analysis of a Mixed Flow Pump Impeller

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Abstract: In order to avoid resonance of a mixed-flow pump impeller and to avoid blade failure due to excessive stress development, it is required to know the natural frequencies at different modes and one should have an idea about the Von Mises stress distribution in the impeller blades. In this present work design and FEM analysis has been carried out on mixed flow pump impeller having different blade positions on the meridional annulus. The natural frequencies at six different modes of the pump impeller were obtained. The maximum Von Mises stress distribution was compared among the impellers having different blade positions. The mixed flow impeller having inlet inclined blade positions on the meridional annulus experiences less amount of Von Mises stress as compared to impeller having trapezoidal blade positions on the meridional annulus. The natural frequencies of the impeller having inlet inclined blade positions on the meridional annulus shows a higher value as compared to compared to impeller having trapezoidal blade positions on the meridional annulus.

Index Terms— Mixed flow pump, Von Mises stress, FEM analysis, natural frequency

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Nomenclature

- \( e \): diameter ratio, dimensionless
- \( g \): acceleration due to gravity, m/sec\(^2\)
- \( H \): pressure head, m
- \( K_w \): velocity coefficient, dimensionless
- \( l \): blade span, mm
- \( N \): rotational speed, rev./min.
- \( P \): power, kW
- \( Q \): volumetric discharge, m\(^3\)/sec.
- \( r \): radius, mm
- \( s \): blade spacing, mm
- \( u_1 \): tangential velocity of blade at inlet, m/sec.
- \( u_2 \): tangential velocity of blade at outlet, m/sec.
- \( V_S \): slip velocity, mm
- \( \alpha \): blade inlet angle, degrees
- \( \alpha_2 \): blade outlet angle, degrees
- \( \alpha_m \): mean blade angle, degrees
- \( \lambda \): blade stagger angle, degrees
- \( \rho \): mass density of the fluid(water), kg/m\(^3\)
- \( \omega \): angular velocity, rad/sec.
- \( \Omega \): dimensionless specific speed, dimensionless
- \( \phi \): semi-cone angle of the impeller, degrees

1. INTRODUCTION

The mixed flow pumps are extensively used in thermal power plants for cooling water duties. The performance of a mixed flow pump can be considerably improved by applying recent advances in understanding the flow behaviour of the pump and the blades. Thus, optimal blade position in the meridional annulus has an important effect on loss and flow deflection. The objective of the blade design is to realize a given velocity triangle with minimum losses as well as minimum stress development in the blade sections.

The industrial design methods are largely based on the application of empirical and semi-empirical rules along with the use of available information in the form of different types of charts and graphs from the existing literature. Impellers are mainly designed using profile...
families which have been developed years ago. The industrial design method often ignores the actual happening within the pump flow passage and is consequently a poor guide when the question of a new design and development of pumps comes to picture. In the above design process the designer has less control than desirable over events. The lack of clear-cut rational basis also inhibits the correction of manufacturing of shortfalls in expected performance. With the rapid advancement of technology the complexity of the problem is also increasing. This scenario demands speedy, efficient and optimal design of a mixed flow pump impeller.

To keep pace with the development and ensure better output from the mixed flow pump, one has to formulate a rational basis for the designing of impellers starting from basic principles, such that the use of empirical correlations is minimized. Such a design from the basic principles has advantages that the designer will have more control over the outcome of his design, while keeping the physical principle constantly in view and enables him to rectify any faults in the performance of the pump.

In this present work, design of a mixed flow pump impeller was carried out for three different blade positioning in the meridional annulus based on basic principles of fluid mechanics and turbo machinery. The above three models were compared on the basis of finite element method (FEM) analysis to select the best among them and for design acceptability.

2. LITERATURE REVIEW

The research and development on mixed flow pump have not gone to an extent as compared to conventional centrifugal and axial flow pumps. The flow through the mixed flow pump is quite complex. The design of the mixed flow pump is much complex because of its complex geometry and large number of flow variables play a very important role in its optimum design. In 1965, Wislicenus [1] initiated the design of a mixed flow pump impeller.


Although, a number of excellent literatures are available on the flow analysis through mixed flow pumps, but study on design and structural analysis of mixed flow pump impellers are scanty. Therefore, the present work is devoted to design and stress analysis of mixed flow pump impeller having different blade positions on the meridional annulus.

3. DESIGN METHODOLOGY

Here the design of the mixed flow pump impeller is based on free vortex theory. The stream surfaces through the meridional annulus are kept parallel to hub and casing, whereas, the hub and casing are parallel to one another. It is also assumes that the meridional velocity distribution remains uniform across the annulus [2]. In the present case a mixed flow pump impeller has been designed for discharge (Q) = 0.125 m³/sec., head developed (H) = 5 m, speed of rotation (N) = 1000 rev/min.

3.1 Design procedure

Calculation of non-dimensional specific speed (Ω)

using the relation,

\[ \Omega = \frac{\omega \sqrt{Q}}{gH^{3/4}} \]  

which results in \( \Omega = 1.998 \) rad./sec.

The inlet diameter (D₁) of the impeller has been calculated using the relation

\[ H = \frac{1}{2gK_n^2} \left[ \frac{\pi D_1 N^2}{e} \right]^2 \]  

Where,

\[ e = \frac{D_1}{D_2} \]  

\[ K_n = \frac{u_2}{\sqrt{2gH}} \]  

The value of \( e \) and \( K_n \) for a given specific speed has been used from A.J. Stepanoff work. [6].
The ideal power requirement is calculated using the relation

\[ P = \frac{Q \rho g H}{1000} kW \]  \(\text{(5)}\)

which results in \( P = 15 \) kW.

The choice of cone angle of the mixed flow pup impeller is chosen as suggested in \([2]\), which must be in between 45° to 60°. In the present case the cone angle has been chosen 60° for the design and a preliminary layout is prepared as shown in fig. 1.

![Fig. 1 Preliminary layout of the blade profile](image)

From the fig. 1, the inlet diameters \((D_{1h}, D_{1t})\) and outlet diameters \((D_{2h}, D_{2t})\) of the impeller were measured. Then the meridional velocity at inlet \( (C_{m1}) \) and outlet \( (C_{m2}) \) were calculated using the relation \( C_m = \frac{Q}{2 \pi r} \). For a proper design, \( C_{m2}/C_{m1} \) should be in between 1.2 to 1.22 \([2]\). So, to keep the value of \( C_{m2}/C_{m1} \) within the specified limit, the rectangular position of the blade in the meridional annulus has been modified, resulting in three different blade positioning in the meridional annulus as shown in fig.2. The inlet and outlet diameters of the impeller blades are shown in Table 1.

![Fig. 2 Final layout of the blade profile](image)

Here, case-me (inlet inclined), case-II (trapezoidal) and case-III (outlet inclined) blade positioning in the meridional annulus were used for further processing. The meridional annulus along the blade span was divided into ten numbers of equal sections parallel to hub and casing.

### Table 1 Blade diameters in mm.

<table>
<thead>
<tr>
<th></th>
<th>Case-I (Inlet inclined)</th>
<th>Case-II (Trapezoidal)</th>
<th>Case-III (outlet inclined)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_{1h})</td>
<td>269.46</td>
<td>294.46</td>
<td>319.46</td>
</tr>
<tr>
<td>(D_{1t})</td>
<td>169.00</td>
<td>169.00</td>
<td>169.00</td>
</tr>
<tr>
<td>(D_{2h})</td>
<td>294.46</td>
<td>319.46</td>
<td>344.46</td>
</tr>
<tr>
<td>(D_{2t})</td>
<td>244.00</td>
<td>244.00</td>
<td>244.00</td>
</tr>
</tbody>
</table>

Calculation of blade angles are carried out using the following relations,

\[ u_1 = \frac{\pi D_1 \rho \omega}{60} \text{ m/sec} \]  \(\text{(6)}\)

\[ u_2 = \frac{\pi D_2 \rho \omega}{60} \text{ m/sec} \]  \(\text{(7)}\)

\[ \tan \alpha_1 = \frac{u_1}{C_{m1}} \]  \(\text{(8)}\)

\[ \tan \alpha_2 = \left( \frac{u_2 - C_{m2} \cos \gamma}{C_{m2} \sin \phi} \right) \]  \(\text{(9)}\)

where,

\[ V_S = \frac{\pi u_2 \cos \alpha_2 \sin \phi}{Z} \]  \(\text{(10)}\)

mean blade angle,

\[ \tan \alpha_m = \left( \frac{\tan \alpha_1 + \tan \alpha_2}{2} \right) \]  \(\text{(11)}\)

pitch,

\[ S = \frac{\pi D_m}{Z} \]  \(\text{(12)}\)

and

\[ D_m = \frac{D_1 + D_2}{2} \]  \(\text{(13)}\)

actual chord,

\[ C = \frac{C'}{\cos \lambda} \]  \(\text{(14)}\)

The blade stagger angle \( \gamma \) has been calculated using Carter's correlation.

The blade angles calculated for each section along the blade span using above relations have to be verified to
satisfy Leiblein blade diffusion factor \( D_t \) to avoid separation of boundary layer from the blade surfaces. The Leiblein blade diffusion factor

\[
D_t = \left[ 1 - \frac{\cos \alpha_1}{\cos \alpha_2} \right] + \left( \frac{S}{C} \right) \frac{\cos \alpha_1}{2} \left( \tan \alpha_1 - \tan \alpha_2 \right) < 0.6 \quad (15)
\]

If the blade angles fail to satisfy the blade diffusion factor, the shifting of the blade positioning along the hub on the meridional annulus has to be made and all the calculation of blade angles to be repeated till all the desired conditions \((\alpha_1 < \alpha_2)\) and \(D_t < 0.6\) are satisfied. Here, NACA 10C4 blade profile is used for the impeller blades as suggested in [5].

### 3.2 Construction of 3D model of the impeller and FEM analysis

Once the blade angles were calculated, the straight two-dimensional cascade was transformed into the corresponding conical cascade using conformal transformation method. The blade sections were staged one over another by maintaining proper stagger angle. Finally, using surface revolution method, 3D model of the mixed flow pump impeller has been prepared. In this work three different blade positioning are selected as shown in fig.2 and corresponding 3D models of the mixed flow pump impeller were prepared. To find out the development of stresses in the above impellers and also to know the natural frequencies, finite element analysis (FEM) analysis has been carried out.

In this present work, the FEM analysis has been carried out using CATIA V5 software to find the development of Von Mises stress, natural frequencies of the mixed flow pump impeller having different blade positioning on the meridional annulus.

### 4. RESULTS AND DISCUSSION

The design parameters such as blade angles, camber angles, stagger angles, chord dimension were calculated at different sections along the blade span for the three different blade positioning in the meridional annulus. During the process of design calculations, it is observed that the case-III failed to meet the basic design criteria \((\alpha_1 > \alpha_2)\) so, designing of the case-III was discarded. Therefore, further designing of case-I and case-II were undertaken. The span wise variation of blade inlet angle \(\alpha_1\), blade outlet angle \(\alpha_2\), blade stagger angle \(\gamma\) for case-I and case-II are shown in fig. 3 and fig.4 respectively.

![Fig. 3 Span wise variation of blade angles for inlet inclined blade position (case-I) on meridional annulus](image)

![Fig. 4 Span wise variation of blade angles for trapezoidal blade position (case-II) on meridional annulus](image)

The blade solidity which is blade spacing by blade chord length \((S/C)\) plays a very important role on the overall performance a turbo machine. So to vary this parameter one needs to vary blade spacing, pitch and chord length for optimal design. In this present work the optimal number of blade was found to be eight. The blade solidity \((S/C)\) is related to coefficient of lift \((C_L)\) for an aerofoil/hydrofoil blade which is given by

\[
C_L = 2\left( \frac{S}{C} \right) (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m \quad (16)
\]

Where, \(\alpha_m\) is the mean blade angle as given as

\[
\tan \alpha_m = \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2) \quad (17)
\]

The variations of blade solidity along blade span (hub to tip) for case-I and case-II are shown in Fig. 5.
Fig. 5 Span wise variation of blade solidity for both inlet inclined blade position (case-I) and trapezoidal blade position (case-II) on meridional annulus.

From the fig.3 and fig. 4, it is observed that variation of blade angles from hub to tip are more for case-II as compared to case-I. More the twisting, the more is the stress development in the blades. This contributes to the more blade loading for the case-II as compared to case-I. From the fig. 5, it is also observed that the variation of blade solidity(S/C) along the blade span is quite smooth and linear in nature for case-I as compared to case-II. This contributes to the better lift coefficient ($C_L$) for case-I and hence, better hydraulic performance. As stated earlier that in this work NACA 10C4 blade profile was used. The cross section of the blades at hub and tip section of case-I and case-II are shown in fig. 6 and fig. 7 respectively.

Once the design of the impeller blades was completed, 3-D model of the impeller was prepared using CATIA V5 solid modeling software. The 3-D model for the case-I and case-II are shown in fig. 8 and fig. 9 respectively.

The structural analysis of the above two models were further carried out for Von Mises stress and natural frequency.

Fig. 6 Cross section of the blades at hub and tip for inlet inclined blade position on meridional annulus (case-I)

Fig. 7 Cross section of the blades at hub and tip for trapezoidal blade position on meridional annulus (case-II)

Fig. 8 3-D Model of the impeller for inlet inclined blade position on meridional annulus (case-I)

Fig. 9 3-D Model of the impeller for trapezoidal blade position on meridional annulus (case-II)

Before proceeding to FEM analysis on the pump impellers, validation of the finite element formulation was checked and grid independency test was carried out to choose the optimum element size for FEM analysis. To do so, a rectangular shaped cantilever beam having the length equivalent to the span of the blade, the width, equivalent to the average width of the blade and the thickness, equivalent to the average thickness of the blade were taken. A normal force equivalent to maximum pressure force, which the impeller blades are supposed to experience, was applied on one side of the cantilever beam. Here, it is assumed that the pressure force is acting on the blades as uniformly distributed load (UDL). Calculation was made for the analytical solution for the Von Mises stress. Using CATIA V5 software, numerical analysis for the same has been carried out.
using seven different sizes of the tetrahedral element namely 3, 3.5, 4, 4.5, 5, 5.5 and 6mm. Among the various element sizes, 4, 4.5 and 5 mm elements showed an excellent agreement between the analytical result and the numerical result. So, 5 mm tetrahedral element was chosen for the FEM analysis for the impellers. The results of the grid independency test are shown in fig. 10. The Von Mises stress distribution for the case-I and case-II are shown in fig. 11 and fig. 12 respectively. From the figs. 11 & 12, it is observed that case-I is experiencing less Von Mises stress as compared to case-II.

Here an attempt has also made to find out natural frequencies of the impeller at first six modes. The natural frequencies of the impeller having two different blade positions, case-I and case-II are tabulated and shown in Table 2. From the natural frequency analysis it is observed that the value of the natural frequencies for the case-I is little higher than that of case-II.

**Table 2** Comparison of natural frequencies for both inlet inclined blade position (case-I) and trapezoidal blade position (case-II) on meridional annulus

<table>
<thead>
<tr>
<th>Mode</th>
<th>Inlet inclined blade position (case-I) (Hz)</th>
<th>Trapezoidal blade position (case-II) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>132.655</td>
<td>131.716</td>
</tr>
<tr>
<td>2</td>
<td>132.692</td>
<td>131.818</td>
</tr>
<tr>
<td>3</td>
<td>249.244</td>
<td>246.816</td>
</tr>
<tr>
<td>4</td>
<td>314.003</td>
<td>315.467</td>
</tr>
<tr>
<td>5</td>
<td>1323.26</td>
<td>1324.54</td>
</tr>
<tr>
<td>6</td>
<td>1323.29</td>
<td>1324.46</td>
</tr>
</tbody>
</table>

**5. CONCLUSION**

A mixed flow pump impeller of non dimensional specific speed of 1.998 rad/sec has been designed for use in large cooling water duties of thermal power plants. From the analysis, it is found that the mixed flow pump impeller having inlet inclined blade position on the meridional annulus (case-I) experiences less Von Mises stress as compared to trapezoidal blade position on meridional annulus (case-II). From the natural frequency analysis it is also found that the value of the natural frequencies for the inlet inclined blade position on meridional annulus (case-I) is more than that in the case of trapezoidal blade position on meridional annulus (case-II). The blade solidity along the span for the inlet inclined blades shows better characteristics than that of the trapezoidal blade position on meridional annulus. Therefore, from this present work, it can be concluded that the mixed flow pump impeller having inlet inclined blade position on meridional annulus can be a better choice for use.

**REFERENCES**


